Reply to "Comment on 'Critique of *q*-entropy for thermal statistics'"

Michael Nauenberg
Department of Physics, University of California, Santa Cruz, California 95064, USA
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It is shown that in his Comment Tsallis did not point out any flaws in the main criticism of my paper, namely, that the *q*-entropy formalism fails to satisfy a fundamental law of thermodynamics. Instead, he presented a numerical simulation of planar rotators with long range interactions which turns out to be irrelevant to my critique.

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In the abstract of his Comment [1], and in a section entitled "Thermal contact between systems with different values of q and the 0th principle of thermodynamics," Tsallis acknowledges [2] that the essential point of my critique [3] is that the q-entropy formalism does not determine the equilibrium temperature for systems with different values of q. In my critique, I paraphrased this crucial failure of Tsallis's formalism by stating that a conventional thermometer satisfying Boltzmann-Gibbs (BG) statistics could not measure the temperature of any supposed q-entropic system with $q \neq 1$, and I concluded that "the laws of thermodynamics would therefore fail to have general validity" unless all systems have the same value of q. Since Tsallis agrees that there are at least *some* systems in thermal equilibrium for which $q=1$, corresponding to BG statistics, my analysis implies that *all* systems in *thermal equilibrium* must satisfy these statistics. But rather than to point out any flaws in my critique, which concerns the *theoretical* basis for the q-entropy formalism, Tsallis has responded by presenting some *numerical* results of a molecular dynamics simulation of coupled planar rotators with long range interactions. It has been known for some time [4] that this system exhibits *quasistationary states* below the transition temperature which do not satisfy BG statistics. Tsallis claims that a smaller system, constructed out of planar rotators but with only nearest-neighbor interactions, which satisfies BG statistics, can measure the temperature of these quasistationary states. The details of Tsallis's simulation are discussed in a separate paper [5], but no evidence is presented that this quasistationary state satisfies the distribution associated with q-entropy. Instead, Tsallis merely asserts that the quasistationary state found in his simulation "*might* (my italics) be described by the q-statistics." But recently it has been shown by Yamaguchi *et al.* [6] that the tails of the momentum distribution for these quasistationary states *do not* satisfy the power law dependence which is predicted by q-statistics. Therefore, Tsallis's simulation is irrelevant to my critique [3]. Finally, I would like to point out that Tsallis's definition of the temperature of the planar rotators as twice their mean kinetic energy is strictly justified only if the model does satisfy BG statistics. In general, the thermodynamic definition of temperature is the inverse of the derivative of the entropy with respect to the energy [7], but neither the entropy nor this derivative was evaluated by Tsallis in his simulation.

In the title of my paper [3], I already indicated that my critique of the q-entropy formalism was confined mainly to its relevance for systems in *thermal* equilibrium. But Tsallis devoted much of his Comment [1] to applications of this formalism to nonthermal problems such as the logistic and standard maps, growth models, small clusters of atoms, and other systems that are not in thermal equilibrium. These applications are not relevant to my critique, and therefore I will not discuss them here. In conclusion, Tsallis's Comment [1] cannot be regarded as a rebuttal to my criticisms [3] of the q-entropy formalism.

[1] C. Tsallis, Phys. Rev. E **69**, 038101 (2004).
[2] In this section Tsallis begins with the remark "We focus now on a strong and crucial statement in Ref. [1] (here Ref. [3]), namely, 'a Boltzmann-Gibbs thermometer would not be able to measure the temperature of a q-entropic system.'"
[3] M. Nauenberg, Phys. Rev. E **67**, 036114 (2003).
[4] Vito Latora, Andrea Rapisarda, and Stefano Ruffo, Phys. Rev. Lett. **83**, 2104 (1999).
[5] Luis G. Moyano, Fulvio Baldovin, and Constantino Tsallis, e-print cond-mat/0305091.
[6] Y. Y. Yamaguchi, J. Barre, F. Bouchet, T. Dauxois, and S. Ruffo, e-print cond-mat/0312480.
[7] In my critique [3] I have shown that this standard definition of temperature cannot be applied in the q-entropy formalism.

